Spillovers, Experiments, and Noncompliance

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The Stable Unit Treament Value Assumption (SUTVA)

In Words

A person's outcome depends only on her *own* treatment.

In Math

$$Y_i = (1 - D_i)Y_{i0} + D_i Y_{i1} = Y_{i0} + D_i\,(Y_{i1} - Y_{i0})$$

Spillovers aka Interference

A person's outcome depends on the treatments of *other people*.

Terminology

Manski calls SUTVA **individualistic treatment response** (ITR).

Spillovers $\implies$ Two Kinds of Causal Effects

Direct

Causal effect of Alice's treatment on *her own* outcome.

Indirect

Causal effect of Alice's treatment on *Bob's* outcome.

Note

Some people say that there is a third effect: the **total** effect. This is just a sum of indirect and direct effects.

Empirical Example with Potential for Indirect Treatment Effects

Crepon et al. (2013; QJE)

The Experiment

- Large-scale job-seeker assistance program in France.
- Randomized offers of intensive job placement services.

Treatment Effects

Direct: If Alice receives job placement, will she be more likely to find a job?

Indirect: If Alice receives job placement, will **Bob** be **less likely** to find a job?

Policy Question

Do the indirect effects of job placement partially or completely offset the direct effects?

Why might we expect indirect effects?

Crepon et al. (2013; QJE)

Displacement Effects of Labor Market Policies

> *"Job seekers who benefit from counseling may be more likely to get a job, but at the expense of other unemployed workers with whom they compete in the labor market. This may be particularly true in the short run, during which vacancies do not adjust: the unemployed who do not benefit from the program could be partially crowded out."*

Potential Outcomes with Spillovers / Interference

Notation

- $\mathbf{D} = (D_1, D_2, \ldots, D_N) \equiv$ vector of treatments of *everyone* in the experiment.
- Use parentheses rather than subscripts to denote potential outcomes.

Fully General Potential Outcomes: $Y_i(\mathbf{D})$

- Alice's outcome may depend on treatments of **everyone in the experiment**.
- If so, then she has 2^N potential outcomes: one for each allocation of treatments.

Clearly this is not going to work...

Partial Interference

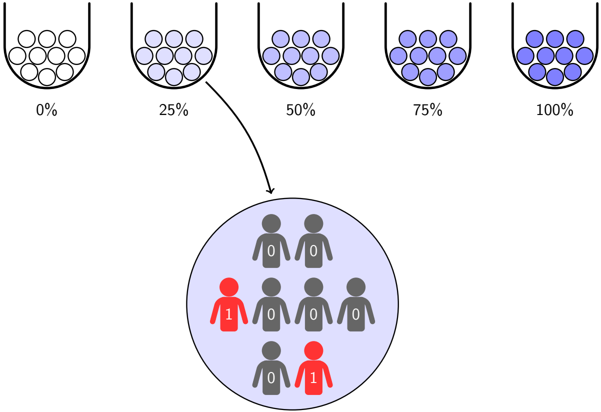
- Problem: way too many potential outcomes!
- Solution: assume only a **subset** of other people affect Alice's outcome.
- E.g. perhaps Alice only experiences spillovers from others in her labor market.
- Suppose that there are G "groups" indexed $g = 1, \ldots, G$
- Each person belongs to a single, known group; (i, g) denotes person i in group g
- $\mathbf{D}_g$ is the vector of treatments of people from group g.
- Partial Interference: $Y_{ig}(\mathbf{D}) = Y_{ig}(\mathbf{D}_g)$
- **Spillovers within but not between groups**.
- Manski calls this "interactions in a reference group."

Anonymous Interactions

- Let N_g be the number of people in group g
- Partial interference reduce's Alice's potential outcomes from 2^N to 2^{N_g}
- But unless N_g is tiny, this number is **still too many for us to make progress**.
- Make another assumption to further reduce the number of potential outcomes.
- Call the other people in Alice's group her **neighbors**.
- **Anonymous Interactions:** only *number* treated neighbors matters, not *identities*.
- Let $\bar{D}_{ig}$ denote the share of Alice's neighbors who are treated.
- Potential Outcome Functions: $Y_{ig}(\mathbf{D}) = Y_{ig}(\mathbf{D}_g) = Y_{ig}(D_{ig}, \bar{D}_{ig})$.

The Randomized Saturation Design

- Two-stage experiment for studying spillovers under partial interference.
  1. Randomly assign **saturations** to groups: share of people treated.
  2. Randomly assign treatments to individuals using group's saturation.
- Hudgens & Halloran (2008)



Identifying the Direct Effect

Notation

Y_{ig}	observed outcome of person (i, g)
$S_g \in \mathcal{S} \subseteq [0,1]$	saturation assigned to group g
$D_{ig} \in \{0,1\}$	treatment assigned to person (i, g)

Random Assignment

S_g and D_{ig} are independent of the potential outcomes.

The Direct Effect: DE(s)

$$
\begin{aligned}
\mathbb{E}[Y_{ig}|D_{ig} = 1, S_g = s] - \mathbb{E}[Y_{ig}|D_{ig} = 0, S_g = s] \\
&= \mathbb{E}[Y_{ig}(1,s)|D_{ig} = 1, S_g = s] - \mathbb{E}[Y_{ig}(0,s)|D_{ig} = 0, S_g = s] \\
&= \mathbb{E}[Y_{ig}(1,s) - Y_{ig}(0,s)] \equiv \mathrm{DE}(s)
\end{aligned}
$$

Identifying the Indirect Effects

The Indirect Effect When Untreated: $\text{IE}_0(s \to s')$

$$
\begin{aligned}
\mathbb{E}[Y_{ig}|D_{ig} = 0, S_g = s'] - \mathbb{E}[Y_{ig}|D_{ig} = 0, S_g = s]& \\
&= \mathbb{E}[Y_{ig}(0, s')|D_{ig} = 0, S_g = s'] - \mathbb{E}[Y_{ig}(0, s)|D_{ig} = 0, S_g = s] \\
&= \mathbb{E}[Y_{ig}(0, s') - Y_{ig}(0, s)] \equiv \text{IE}_0(s \to s')
\end{aligned}
$$

The Indirect Effect When Treated: $\text{IE}_1(s \to s')$

$$
\begin{aligned}
\mathbb{E}[Y_{ig}|D_{ig} = 1, S_g = s'] - \mathbb{E}[Y_{ig}|D_{ig} = 1, S_g = s]& \\
&= \mathbb{E}[Y_{ig}(1, s')|D_{ig} = 1, S_g = s'] - \mathbb{E}[Y_{ig}(1, s)|D_{ig} = 1, S_g = s] \\
&= \mathbb{E}[Y_{ig}(1, s') - Y_{ig}(1, s)] \equiv \text{IE}_0(s \to s')
\end{aligned}
$$

Non-compliance in Randomized Saturation Experiments

DiTraglia et al (2022)

Identification

Beyond Intent-to-Treat: Direct & indirect causal effects under 1-sided non-compliance.

Estimation

Simple, asymptotically normal estimator under large/many-group asymptotics.

Application

French labor market experiment: Crepon et al. (2013; QJE)

Notation

Sample Size and Indexing

Groups	$g = 1, \ldots G$
Individuals in g	$i = 1, \ldots, N_g$

Observables

Y_{ig}	outcome of (i, g)
$Z_{ig} \in \{0, 1\}$	treatment offer to (i, g)
$D_{ig} \in \{0, 1\}$	treatment take-up of (i, g)
$\bar{D}_{ig} \in [0, 1]$	take-up of (i, g)'s "neighbors"
$S_g \in \mathcal{S} \subseteq [0, 1]$	saturation of group g

Overview of Assumptions

(i) Experimental Design: Randomized Saturation ✓

(ii) Potential Outcomes: Correlated Random Coefficients Model

(iii) Treatment Take-up: 1-sided Noncompliance & "Individualized Offer Response"

(iv) Exclusion Restriction for (Z_{ig}, S_g)

(v) Rank Condition

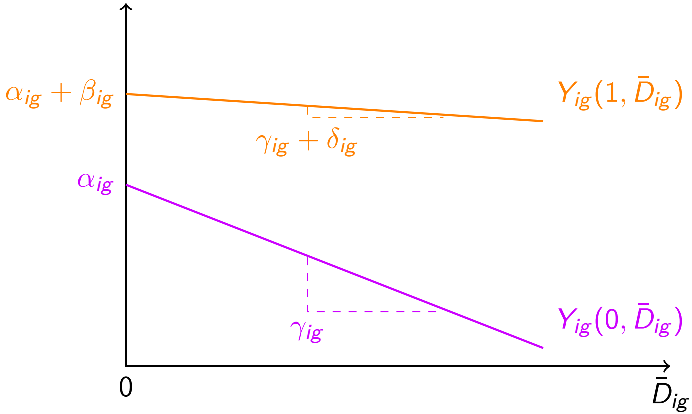
Assumption (ii) – Correlated Random Coefficients (CRC) Model

$$Y_{ig}(\boldsymbol{D}) = Y_{ig}(\boldsymbol{D}_g) = Y_{ig}(D_{ig}, \bar{D}_{ig}) = \mathbf{f}(\bar{D}_{ig})' \left[(1 - D_{ig})\boldsymbol{\theta}_{ig} + D_{ig}\boldsymbol{\psi}_{ig}\right]$$

$\mathbf{f}$	vector of known functions	Lipschitz continuous on $[0, 1]$
$(\boldsymbol{\theta}_{ig}, \boldsymbol{\psi}_{ig})$	random variables	possibly dependent on $(D_{ig}, \bar{D}_{ig})$

This Talk – linear potential outcomes model...

$$Y_{ig}(D_{ig}, \bar{D}_{ig}) = \alpha_{ig} + \beta_{ig} D_{ig} + \gamma_{ig} \bar{D}_{ig} + \delta_{ig} D_{ig} \bar{D}_{ig}$$



Indirect Effects

Treated: $\gamma_{ig} + \delta_{ig}$

Untreated: γ_{ig}

Direct Effects

$\beta_{ig} + \delta_{ig} \bar{D}_{ig}$

Assumption (iii) – Treatment Take-up

1-sided Non-compliance

Only those offered treatment can take it up.

Individualistic Offer Response (IOR)

$$D_{ig}(\boldsymbol{Z}) = D_{ig}(\boldsymbol{Z}_g) = D_{ig}(Z_{ig})$$

Notation

$C_{ig} = 1$ iff (i, g) is a complier; $\bar{C}_{ig} \equiv$ share of compliers among (i, g)'s neighbors.

$$\boxed{(\text{IOR}) + (\text{1-Sided}) \Rightarrow D_{ig} = C_{ig} \times Z_{ig}}$$

No Evidence Against IOR in Our Example

Data from Crepon et al. (2013; QJE)

(IOR) + (1-Sided)

Take-up only depends on *own* offer:

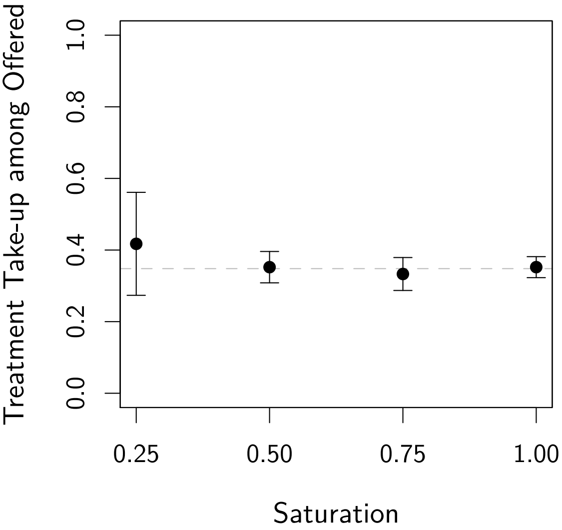
$D_{ig} = C_{ig} \times Z_{ig}$

Testable Implication

$\mathbb{E}[D_{ig}|Z_{ig} = 1, S_g] = \mathbb{E}[D_{ig}|Z_{ig} = 1]$

Figure at right

Take-up among offered doesn't vary with saturation (p = 0.62)



Assumption (iv) – Exclusion Restriction

Notation

$\mathbf{B}_g$	random coefficients for everyone in group g
$\boldsymbol{C}_g$	complier indicators for everyone in group g
$\boldsymbol{Z}_g$	treatment offers for everyone in group g

Exclusion Restriction

(i) $S_g \perp\!\!\!\perp (\boldsymbol{C}_g, \mathbf{B}_g, N_g)$

(ii) $\boldsymbol{Z}_g \perp\!\!\!\perp (\boldsymbol{C}_g, \mathbf{B}_g) | (S_g, N_g)$

Näive IV Does Not Identify the Spillover Effect

Unoffered Individuals

$$\begin{aligned} Y_{ig} &= \alpha_{ig} + \cancel{\beta_{ig} D_{ig}} + \gamma_{ig} \bar{D}_{ig} + \cancel{\delta_{ig} D_{ig} \bar{D}_{ig}} \\ &= \underbrace{\mathbb{E}[\alpha_{ig}]}_{\alpha} + \underbrace{\mathbb{E}[\gamma_{ig}]}_{\gamma} \bar{D}_{ig} + \underbrace{(\alpha_{ig} - \mathbb{E}[\alpha_{ig}]) + (\gamma_{ig} - \mathbb{E}[\gamma_{ig}]) \bar{D}_{ig}}_{\varepsilon_{ig}} \end{aligned}$$

IV Estimand

$$\gamma_{IV} = \frac{\text{Cov}(Y_{ig}, S_g)}{\text{Cov}(\bar{D}_{ig}, S_g)} = \gamma + \frac{\text{Cov}(\varepsilon_{ig}, S_g)}{\text{Cov}(\bar{D}_{ig}, S_g)} = \ldots = \gamma + \frac{\text{Cov}(\gamma_{ig}, \bar{C}_{ig})}{\mathbb{E}(\bar{C}_{ig})}$$

Identification – Average Spillover Effect when Untreated

One-sided Noncompliance

$$(1 - Z_{ig})Y_{ig} = (1 - Z_{ig})(\alpha_{ig} + \cancel{\beta_{ig} D_{ig}} + \gamma_{ig}\bar{D}_{ig} + \cancel{\delta_{ig} D_{ig}\bar{D}_{ig}}) = (1 - Z_{ig})\begin{pmatrix} 1 \\ \bar{D}_{ig} \end{pmatrix}' \begin{pmatrix} \alpha_{ig} \\ \gamma_{ig} \end{pmatrix}$$

Theorem

$(Z_{ig}, \bar{D}_{ig}) \perp\!\!\!\perp (\alpha_{ig}, \gamma_{ig}) | (\bar{C}_{ig}, N_g)$.

$$\begin{aligned}
\mathbb{E}\left[\begin{pmatrix} 1 \\ \bar{D}_{ig} \end{pmatrix} (1 - Z_{ig})Y_{ig} \middle| \bar{C}_{ig}, N_g \right] &= \mathbb{E}\left[(1 - Z_{ig}) \begin{pmatrix} 1 & \bar{D}_{ig} \\ \bar{D}_{ig} & \bar{D}_{ig}^2 \end{pmatrix} \begin{pmatrix} \alpha_{ig} \\ \gamma_{ig} \end{pmatrix} \middle| \bar{C}_{ig}, N_g \right] \\
&= \underbrace{\mathbb{E}\left[(1 - Z_{ig}) \begin{pmatrix} 1 & \bar{D}_{ig} \\ \bar{D}_{ig} & \bar{D}_{ig}^2 \end{pmatrix} \middle| \bar{C}_{ig}, N_g \right]}_{\equiv \mathbf{Q}_0(\bar{C}_{ig}, N_g)} \mathbb{E}\left[\begin{pmatrix} \alpha_{ig} \\ \gamma_{ig} \end{pmatrix} \middle| \bar{C}_{ig}, N_g \right]
\end{aligned}$$

Average Spillover, Untreated: $\mathbb{E}[Y_{ig}(0,\bar{d})] = \mathbb{E}(\alpha_{ig}) + \mathbb{E}(\gamma_{ig})\bar{d}$

$$\begin{bmatrix} \mathbb{E}(\alpha_{ig}) \\ \mathbb{E}(\gamma_{ig}) \end{bmatrix} = \mathbb{E}\left[\mathbf{Q}_0(\bar{C}_{ig}, N_g)^{-1} \begin{pmatrix} 1 \\ \bar{D}_{ig} \end{pmatrix} (1 - Z_{ig}) Y_{ig}\right]$$

$$\mathbf{Q}_0(\bar{C}_{ig}, N_g) \equiv \mathbb{E}\left[(1 - Z_{ig}) \begin{pmatrix} 1 & \bar{D}_{ig} \\ \bar{D}_{ig} & \bar{D}_{ig}^2 \end{pmatrix} \middle| \bar{C}_{ig}, N_g\right]$$

$\mathbf{Q}_0$ is a *known function*

Distribution of $\bar{D}_{ig}|(\bar{C}_{ig}, N_g)$ determined by experimental design.

Rank Condition: $Y_{ig}(D_{ig}, \bar{D}_{ig}) = \mathbf{f}(\bar{D}_{ig})' \left[(1 - D_{ig})\,\boldsymbol{\theta}_{ig} + D_{ig}\boldsymbol{\psi}_{ig}\right]$

$$\boxed{\mathbf{Q}_z(\bar{c}, n) \equiv \mathbb{E}\left[\mathbb{1}(Z_{ig} = z)\mathbf{f}(\bar{D}_{ig})\mathbf{f}(\bar{D}_{ig})' \middle| \bar{C}_{ig} = \bar{c}, N_g = n\right], \quad z = 0, 1}$$

Rank Condition

$\mathbf{Q}_0(\bar{c}, n), \mathbf{Q}_1(\bar{c}, n)$ invertible for all $(\bar{c}, n)$ in the support of $(\bar{C}_{ig}, N_g)$.

E.g. Linear Model

$$\mathbf{Q}_0(\bar{c}, n) = \begin{bmatrix} \mathbb{E}\{1 - S_g\} & \bar{c}\,\mathbb{E}\{S_g(1 - S_g)\} \\ \bar{c}\,\mathbb{E}\{S_g(1 - S_g)\} & \bar{c}^2\,\mathbb{E}\left\{S_g^2(1 - S_g)\right\} + \frac{\bar{c}}{n-1}\mathbb{E}\left\{S_g(1 - S_g)^2\right\} \end{bmatrix}$$

$$\mathbf{Q}_1(\bar{c}, n) = \begin{bmatrix} \mathbb{E}\{S_g\} & \bar{c}\,\mathbb{E}\left\{S_g^2\right\} \\ \bar{c}\,\mathbb{E}\left\{S_g^2\right\} & \bar{c}^2\,\mathbb{E}\left\{S_g^3\right\} + \frac{\bar{c}}{n-1}\mathbb{E}\left\{S_g^2(1 - S_g)\right\} \end{bmatrix}$$

(Rank Condition) + (Assumptions i–iv) $\Rightarrow$ Point Identified Effects

Spillover

$\bar{D}_{ig} \rightarrow Y_{ig}$ for the population, holding $D_{ig} = 0$.

Direct Effect on the Treated

$D_{ig} \rightarrow Y_{ig}$ for compliers as a function of $\bar{d}$.

Indirect Effects on the Treated

$\bar{D}_{ig} \rightarrow Y_{ig}$ for compliers holding $D_{ig} = 0$ or $D_{ig} = 1$.

Indirect Effect on the Untreated

$\bar{D}_{ig} \rightarrow Y_{ig}$ for never-takers holding $D_{ig} = 0$.

Feasible Estimation: Just-Identified IV with "Generated" Instruments

$$\widehat{\boldsymbol{\vartheta}} \equiv \left(\sum_{g=1}^{G} \sum_{i=1}^{N_g} \widehat{\boldsymbol{\mathcal{Z}}}_{ig} \mathbf{X}'_{ig} \right)^{-1} \left(\sum_{g=1}^{G} \sum_{i=1}^{N_g} \widehat{\boldsymbol{\mathcal{Z}}}_{ig} Y_{ig} \right), \quad \widehat{C}_{ig} \equiv \bar{D}_{ig} / \bar{Z}_{ig}$$

Example From Above: $Y_{ig} = \alpha + \gamma \bar{D}_{ig} + \varepsilon_{ig}$

$$\widehat{\boldsymbol{\vartheta}} = \begin{bmatrix} \widehat{\alpha} \\ \widehat{\gamma} \end{bmatrix}, \quad \mathbf{X}_{ig} = \begin{bmatrix} 1 \\ \bar{D}_{ig} \end{bmatrix}, \quad \widehat{\boldsymbol{\mathcal{Z}}}_{ig} = (1 - Z_{ig}) \mathbf{Q}_0 (\widehat{C}_{ig}, N_g)^{-1} \begin{bmatrix} 1 \\ \bar{D}_{ig} \end{bmatrix}$$

$$\mathbf{Q}_0(\bar{c}, n) = \begin{bmatrix} \mathbb{E}\left\{1 - S_g\right\} & \bar{c}\, \mathbb{E}\left\{S_g(1 - S_g)\right\} \\ \bar{c}\, \mathbb{E}\left\{S_g(1 - S_g)\right\} & \bar{c}^2\, \mathbb{E}\left\{S_g^2(1 - S_g)\right\} + \frac{\bar{c}}{n-1} \mathbb{E}\left\{S_g(1 - S_g)^2\right\} \end{bmatrix}$$

Crepon Example: Labor Market Displacement Effects

(SEs clustered at labor market level)

$\mathbb{E}[\gamma_{ig}\vert\text{Type}]$	Popn.	Never-takers	Compliers
$\mathbb{P}$(Long-term Employment)	-0.09	0.14	-0.51
	(0.07)	(0.09)	(0.24)
$\mathbb{P}$(Any Employment)	-0.11	0.14	-0.56
	(0.06)	(0.09)	(0.24)

$$\mathbb{E}[Y_{ig}(0, \bar{d})|\text{Type}] = \mathbb{E}[\alpha_{ig}|\text{Type}] + \mathbb{E}[\gamma_{ig}|\text{Type}] \times \bar{d}$$

Crepon Example: Protective Effect of Treatment for Compliers

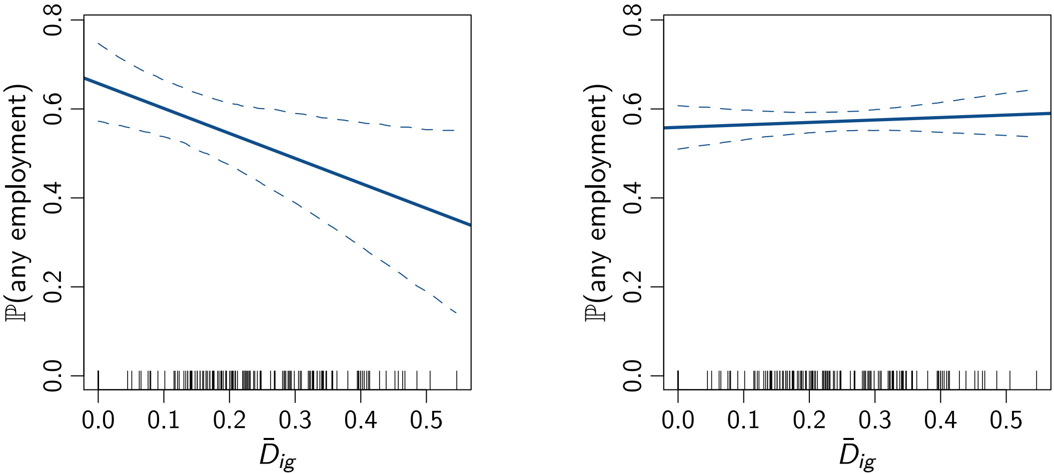
(SEs clustered at labor market level)

	α^c	γ^c	β^c	δ^c
$\mathbb{P}$(Long-term Employment)	0.48	-0.51	-0.09	0.62
	(0.04)	(0.24)	(0.05)	(0.25)
$\mathbb{P}$(Any Employment)	0.66	-0.56	-0.10	0.62
	(0.04)	(0.24)	(0.05)	(0.25)

$$\mathbb{E}[Y_{ig}(0, \bar{d})|\text{Complier}] = \alpha^c + \gamma^c \cdot \bar{d}$$

$$\mathbb{E}[Y_{ig}(1, \bar{d})|\text{Complier}] = (\alpha^c + \beta^c) + (\gamma^c + \delta^c) \times \bar{d}$$

Untreated: $\mathbb{E}[Y_{ig}(0, \bar{d})|\text{Complier}]$ Treated: $\mathbb{E}[Y_{ig}(1, \bar{d})|\text{Complier}]$



Conclusion

Identification

Go beyond ITTs to identify average direct and indirect effects in randomized saturation experiments with 1-sided non-compliance.

Estimation

Simple asymptotically normal estimator under large/many-group asymptotics.

Application

Negative spillovers for those willing to take up the program offset by positive direct treatment effects: selection on gains.