Lecture 1 - Potential Outcomes and Selection Bias

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Treatment Effects: The Basics

Treatment Effects / Causal Inference – "What If?"

- **Outcome Variable:** Y
- **Treatment Variable:** $D = 0$ if untreated and $D = 1$ if treated
- **Treatment Effect:** if we changed D from 0 to 1, how would Y change?
- **Example:** returns to education

Important!

A treatment effect is a *hypothetical* within-person comparison at a particular moment in time, always involving a *counterfactual*.

Figure 1: Would Alice earn a higher wage if she attended university?

Potential Outcomes Framework

- Imagine that each person has two **potential outcomes**: (Y_0, Y_1).
- This is a thought experiment: we only see the **observed outcome** Y.
- Treatment variable determines which potential outcome is observed:
  - $D = 0 \Rightarrow Y = Y_0$
  - $D = 1 \Rightarrow Y = Y_1$
- Only one of (Y_0, Y_1) is observed for any given person at any given time.
- Unobserved potential outcome is a **counterfactual**, i.e. a **what if?**

Figure 2: Alice earns Y_1 if she attends university, Y_0 if she doesn't attend. Her treatment effect is $(Y_1 - Y_0)$.

What is this course about?

Treatment Effect: $(Y_1 - Y_0)$

A hypothetical, within-person comparison of potential outcomes.

Challenges

1. **Heterogeneous Treatment Effects**
   - Different people probably have different potential outcomes.
   - If so, they likely have different treatment effects.
2. **Fundamental Problem of Causal Inference**
   - We can never observe both Y_0 and Y_1 for the same person at the same time.
   - So, in practice, we can only make *between person* comparisons.
3. **Selection**
   - It matters how D is assigned: who gets treated and why?
   - Perhaps people *know* their treatment effects and *choose* to get treated.

Example with Heterogeneous Treatment Effects: (Potentially) Fatal Disease

	Y_0	Y_1	$(Y_1 - Y_0)$
Doomed	0	0	0
Cured	0	1	1
Allergic	1	0	-1
Immune	1	1	0

Figure 3: Four different kinds of people: all possible combinations of potential outcomes when Y is binary.

- Outcome: $Y = 1$ survive; $Y = 0$ die.
- Treatment: $D = 1$ take drug, $D = 0$ don't take drug.
- Helps **Cured**, harms **Allergic**, no effect on **Doomed** / **Immune**

An omniscient being has told you the shares of Doomed, Allergic, Immune.

Exercise: What is the average value of $\Delta \equiv (Y_1 - Y_0)$?

	Y_0	Y_1	$(Y_1 - Y_0)$	Share
Doomed	0	0	0	20%
Cured	0	1	1	20%
Allergic	1	0	-1	40%
Immune	1	1	0	20%

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Exercise: What is the average value of $\Delta \equiv (Y_1 - Y_0)$?

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Cured	0	1	1	20%
Allergic	1	0	-1	40%
Immune	1	1	0	20%

Solution: By the definition of Expected Value $\mathbb{E}(\Delta) = -0.2$.

$$\begin{aligned}\mathbb{E}(\Delta) &= -1 \times \mathbb{P}(\Delta = -1) + 0 \times \mathbb{P}(\Delta = 0) + 1 \times \mathbb{P}(\Delta = 1) \\ &= \mathbb{P}(\Delta = 1) - \mathbb{P}(\Delta = -1) \\ &= \mathbb{P}(\text{Cured}) - \mathbb{P}(\text{Allergic})\end{aligned}$$

What can we learn *without* the help of an omniscient being?

Fundamental Problem of Causal Inference

Only observe one of the potential outcomes (Y_0, Y_1) for any individual.

	D	Y	Y_0	Y_1	$(Y_1 - Y_0)$
Amir	1	1	?	1	?
Brooke	0	0	0	?	?
Carlos	0	1	1	?	?
Derek	1	1	?	1	?
Elise	0	1	1	?	?
Felix	1	1	?	1	?
Gabriela	0	1	1	?	?
Hannah	0	0	0	?	?
Issac	0	1	1	?	?
Jasmine	1	1	?	1	?

Exercise: What *can't* we learn?

1. Amir's individual treatment effect.
2. The fraction of people with a positive treatment effect.
3. The variance of treatment effects.
4. The average treatment effect.

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Derek	1	1	?	1	?
Elise	0	1	1	?	?
Felix	1	1	?	1	?
Gabriela	0	1	1	?	?
Hannah	0	0	0	?	?
Issac	0	1	1	?	?
Jasmine	1	1	?	1	?

Exercise: What *can't* we learn?

1. Amir's individual treatment effect.
2. The fraction of people with a positive treatment effect.
3. The variance of treatment effects.
4. The average treatment effect.

Solution: We can never learn 1, 2, or 3.

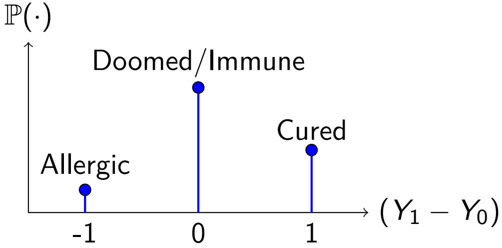
We never observe both Y_0 and Y_1 for the same person at the same time.

Can't learn any of these:

- Individual treatment effects
- Joint distribution of (Y_0, Y_1), i.e. how they co-vary
- Distribution of individual treatment effects $(Y_1 - Y_0)$
- Share of people who benefit from the treatment.

		Y_1 = 0	Y_1 = 1
Y_0	0	$\mathbb{P}$(Doomed)	$\mathbb{P}$(Cured)
	1	$\mathbb{P}$(Allergic)	$\mathbb{P}$(Immune)

Figure 4: Joint Distribution of (Y_0, Y_1)



The Average Treatment Effect (ATE) is *Very Special*

Exercise: Why can't we learn $\text{Var}(Y_1 - Y_0)$?

The Average Treatment Effect (ATE) is *Very Special*

Exercise: Why can't we learn $\text{Var}(Y_1 - Y_0)$?

Depends on the joint distribution of (Y_0, Y_1) through $\text{Cov}(Y_0, Y_1)$

$$\text{Var}(Y_1 - Y_0) = \text{Var}(Y_1) + \text{Var}(Y_0) - 2\text{Cov}(Y_0, Y_1).$$

What's special about $\mathbb{E}(Y_1 - Y_0)$?

- **Linearity of Expectation:** $\mathbb{E}(cW) = c\mathbb{E}(W), \quad \mathbb{E}(W + Z) = \mathbb{E}(W) + \mathbb{E}(Z).$
- Implication: $\mathbb{E}(Y_1 - Y_0) = \mathbb{E}(Y_1) - \mathbb{E}(Y_0)$ regardless of joint distribution!
- Between-person comparison replaces infeasible within-person comparison.

Randomized Experiments: Basic Intuition

	D	Y	Y_0	Y_1	$(Y_1 - Y_0)$
Amir	1	1	?	1	?
Brooke	0	0	0	?	?
Carlos	0	1	1	?	?
Derek	1	1	?	1	?
Elise	0	1	1	?	?
Felix	1	1	?	1	?
Gabriela	0	1	1	?	?
Hannah	0	0	0	?	?
Issac	0	1	1	?	?
Jasmine	1	1	?	1	?

- Suppose we flipped a coin to assign D
- Then the observed Y_0 values are a representative sample of all Y_0 values.
- Similarly, the observed Y_1 values are a representative sample of all Y_1 values.
- Difference of sample mean outcomes is a good estimator of ATE $= \mathbb{E}(Y_1) - \mathbb{E}(Y_0)$.
- Between-person comparison replaces infeasible within-person comparison!

Exercise: Calculate $(\bar{Y}_1 - \bar{Y}_0)$

Randomized Experiments: Basic Intuition

	D	Y	Y_0	Y_1	$(Y_1 - Y_0)$
Amir	1	1	?	1	?
Brooke	0	0	0	?	?
Carlos	0	1	1	?	?
Derek	1	1	?	1	?
Elise	0	1	1	?	?
Felix	1	1	?	1	?
Gabriela	0	1	1	?	?
Hannah	0	0	0	?	?
Issac	0	1	1	?	?
Jasmine	1	1	?	1	?

- Suppose we flipped a coin to assign D
- Then the observed Y_0 values are a representative sample of all Y_0 values.
- Similarly, the observed Y_1 values are a representative sample of all Y_1 values.
- Difference of sample mean outcomes is a good estimator of ATE $= \mathbb{E}(Y_1) - \mathbb{E}(Y_0)$.
- Between-person comparison replaces infeasible within-person comparison!

Exercise: Calculate $(\bar{Y}_1 - \bar{Y}_0)$

$$\bar{Y}_1 - \bar{Y}_0 = \frac{1}{4}(1+1+1+1) - \frac{1}{6}(0+1+1+1+0+1) = 1 - \frac{2}{3} = \frac{1}{3}$$

Review of what we've covered so far[1]

- Binary **Treatment** $D \in \{0,1\}$ and **Observed Outcome** Y
- Y depends on **Potential Outcomes** (Y_0, Y_1) via

$$Y = (1-D)Y_0 + DY_1 = Y_0 + D(Y_1 - Y_0)$$

- Only one of (Y_0, Y_1) is observed for each person.
- Unobserved potential outcome: **counterfactual** / **what if?**
- Can't learn quantities that depend on joint distribution of (Y_0, Y_1).
- **Average Treatment Effect:** $\text{ATE} \equiv \mathbb{E}(Y_1 - Y_0) = \mathbb{E}(Y_1) - \mathbb{E}(Y_0)$
- ATE doesn't depend on joint distribution; can learn it from randomized experiment

[1] Video on potential outcomes: https://expl.ai/QHUAVRV

Our Biggest Challenge: Selection into Treatment (aka Counfounding)

Observational Data

Data that do not come from a randomized controlled experiment.

Selection into Treatment

D not randomly assigned $\Rightarrow$ treated and untreated likely differ many other ways, **particularly if people can choose** D.

Confounder

Factor that influences both outcomes Y and whether or not people are treated D.

Figure 5: Selection into treatment

Our Biggest Challenge: Selection into Treatment (aka Counfounding)

Smoking & Cancer

Smokers may be less health conscious in general than non-smokers.

Returns to Education

Oxford undergraduates are on average richer and have higher levels of academic preparation before university.

Voting Behavior

Fox News viewers are older and more likely to live in rural rather than urban areas.

Figure 6: Selection into treatment

Our Biggest Challenge: Selection into Treatment (aka Confounding)

Why not randomize everything?

Randomization may be impossible, impractical or unethical.

Smoking & Cancer

Unethical to randomly assign some people to smoke two packs of cigaretts per day.

Returns to Education

I'd be fired if I randomly admitted some students to Oxford and rejected the rest.

Voting Behavior

Can't force some people to watch Fox news and keep track of how they voted.

Figure 7: Selection into treatment

Selection into Treatment in the Disease Example

	D	Y	Y_0	Y_1	$(Y_1 - Y_0)$
Amir	1	1	0	1	1
Brooke	0	0	0	0	0
Carlos	0	1	1	0	-1
Derek	1	1	0	1	1
Elise	0	1	1	0	-1
Felix	1	1	1	1	0
Gabriela	0	1	1	0	-1
Hannah	0	0	0	0	0
Issac	0	1	1	0	-1
Jasmine	1	1	1	1	0

Population of 10 People

- 2 **Doomed**: Brooke & Hannah
- 2 **Cured**: Amir & Derek
- 4 **Allergic**: Carlos, Elise, Gabriela & Issac
- 2 **Immune**: Felix & Jasmine

Exercise

Calculate the ATE and compare it to $(\bar{Y}_1 - \bar{Y}_0)$.

Selection into Treatment in the Disease Example

	D	Y	Y_0	Y_1	$(Y_1 - Y_0)$
Amir	1	1	0	1	1
Brooke	0	0	0	0	0
Carlos	0	1	1	0	-1
Derek	1	1	0	1	1
Elise	0	1	1	0	-1
Felix	1	1	1	1	0
Gabriela	0	1	1	0	-1
Hannah	0	0	0	0	0
Issac	0	1	1	0	-1
Jasmine	1	1	1	1	0

Population of 10 People

- 2 **Doomed**: Brooke & Hannah
- 2 **Cured**: Amir & Derek
- 4 **Allergic**: Carlos, Elise, Gabriela & Issac
- 2 **Immune**: Felix & Jasmine

Exercise

Calculate the ATE and compare it to $(\bar{Y}_1 - \bar{Y}_0)$.

Solution

- $\text{ATE} = \frac{1}{10}(1 - 1 + 1 - 1 - 1 - 1) = -0.2$
- $(\bar{Y}_1 - \bar{Y}_0) \approx 0.33$

Selection into Treatment in the Disease Example

	D	Y	Y_0	Y_1	$(Y_1 - Y_0)$
Amir	1	1	0	1	1
Brooke	0	0	0	0	0
Carlos	0	1	1	0	-1
Derek	1	1	0	1	1
Elise	0	1	1	0	-1
Felix	1	1	1	1	0
Gabriela	0	1	1	0	-1
Hannah	0	0	0	0	0
Issac	0	1	1	0	-1
Jasmine	1	1	1	1	0

Population of 10 People

- 2 **Doomed**: Brooke & Hannah
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- 2 **Immune**: Felix & Jasmine

Exercise

Why does ATE $= -0.2$ when $\bar{Y}_1 - \bar{Y}_0 \approx 0.33$?

Selection into Treatment in the Disease Example

	D	Y	Y_0	Y_1	$(Y_1 - Y_0)$
Amir	1	1	0	1	1
Brooke	0	0	0	0	0
Carlos	0	1	1	0	-1
Derek	1	1	0	1	1
Elise	0	1	1	0	-1
Felix	1	1	1	1	0
Gabriela	0	1	1	0	-1
Hannah	0	0	0	0	0
Issac	0	1	1	0	-1
Jasmine	1	1	1	1	0

Population of 10 People

- 2 **Doomed**: Brooke & Hannah
- 2 **Cured**: Amir & Derek
- 4 **Allergic**: Carlos, Elise, Gabriela & Issac
- 2 **Immune**: Felix & Jasmine

Exercise

Why does ATE $= -0.2$ when $\bar{Y}_1 - \bar{Y}_0 \approx 0.33$?

Solution

- **Cured** and **Immune** always take the drug
- **Allergic** and **Doomed** never take the drug

Conditional Expectation: $\mathbb{E}(W|X = x)$

Intuition

Average W among everyone with $X = x$, e.g. $\mathbb{E}(\text{Wage}|\text{Attended University } x)$.

Definition

$$\mathbb{E}(W|X = x) \equiv \sum_{\text{all } w} w \cdot \mathbb{P}(W = w|X = x)$$

Linearity

$$\mathbb{E}(cW|X = x) = c\mathbb{E}(W|X = x)$$

$$\mathbb{E}(W + Z|X = x) = \mathbb{E}(W|X = x) + \mathbb{E}(Z|X = x)$$

Counterfactuals and Conditional Expectations

Actual: what happened to the untreated?

$\mathbb{E}(Y|D=0) = \mathbb{E}(Y_0|D=0)$: average outcome actually experienced by untreated.

Actual: what happened to the treated?

$\mathbb{E}(Y|D=1) = \mathbb{E}(Y_1|D=1)$: average outcome actually experienced by treated.

Counterfactual: what if they had been treated?

$\mathbb{E}(Y_1|D=0)$: avg. Y untreated **would have experienced if they had been treated.**

Counterfactual: what if they hadn't been treated?

$\mathbb{E}(Y_0|D=1)$ avg. Y treated **would have experienced if they hadn't been treated.**

A Fundamental Decomposition[2]

$$\underbrace{\mathbb{E}(Y|D=1) - \mathbb{E}(Y|D=0)}_{\text{Observed Difference of Means}} = \mathbb{E}(Y_1|D=1) - \mathbb{E}(Y_0|D=0)$$

$$= \mathbb{E}(Y_1|D=1) - \mathbb{E}(Y_0|D=0) + \mathbb{E}(Y_0|D=1) - \mathbb{E}(Y_0|D=1)$$

$$= \underbrace{\mathbb{E}(Y_1 - Y_0|D=1)}_{\text{TOT}} + \underbrace{[\mathbb{E}(Y_0|D=1) - \mathbb{E}(Y_0|D=0)]}_{\text{Selection Bias}}$$

Treatment on the Treated (TOT)

Average treatment effect for the **kind of people who actually get treated**.

Selection Bias

Compares **counterfactual** outcome for the treated to **actual** outcome for the untreated.

[2]Video: https://expl.ai/DWVNRZU

How might selection matter?

$$\underbrace{\mathbb{E}(Y|D=1) - \mathbb{E}(Y|D=0)}_{\text{Observed Difference of Means}} = \underbrace{\mathbb{E}(Y_1 - Y_0|D=1)}_{\text{TOT}} + \underbrace{[\mathbb{E}(Y_0|D=1) - \mathbb{E}(Y_0|D=0)]}_{\text{Selection Bias}}$$

Returns to Education

Y = Wage; $D = 1$ attend Oxford; Y_0 = wage if you attend; Y_1 = wage if you don't

Selection-on-gains

If people who benefit most are those who choose to attend Oxford then TOT $>$ ATE.

Selection Bias

People who attend Oxford are richer and have higher levels of academic preparation. If these factors are positively related to Y_0 then (Selection Bias) > 0.

TOT and Selection Bias in Disease Example

	D	Y	Y_0	Y_1	$(Y_1 - Y_0)$
Amir	1	1	0	1	1
Brooke	0	0	0	0	0
Carlos	0	1	1	0	-1
Derek	1	1	0	1	1
Elise	0	1	1	0	-1
Felix	1	1	1	1	0
Gabriela	0	1	1	0	-1
Hannah	0	0	0	0	0
Issac	0	1	1	0	-1
Jasmine	1	1	1	1	0

Exercise: Calculate TOT & Selection Bias

TOT and Selection Bias in Disease Example

	D	Y	Y_0	Y_1	$(Y_1 - Y_0)$
Amir	1	1	0	1	1
Brooke	0	0	0	0	0
Carlos	0	1	1	0	-1
Derek	1	1	0	1	1
Elise	0	1	1	0	-1
Felix	1	1	1	1	0
Gabriela	0	1	1	0	-1
Hannah	0	0	0	0	0
Issac	0	1	1	0	-1
Jasmine	1	1	1	1	0

Exercise: Calculate TOT & Selection Bias

$$\begin{aligned} \text{TOT} &\equiv \mathbb{E}(Y_1 - Y_0 | D = 1) \\ &= \frac{1}{4}(1 + 1 + 0 + 0) = 0.5 \end{aligned}$$

TOT and Selection Bias in Disease Example

	D	Y	Y_0	Y_1	$(Y_1 - Y_0)$
Amir	1	1	0	1	1
Brooke	0	0	0	0	0
Carlos	0	1	1	0	-1
Derek	1	1	0	1	1
Elise	0	1	1	0	-1
Felix	1	1	1	1	0
Gabriela	0	1	1	0	-1
Hannah	0	0	0	0	0
Issac	0	1	1	0	-1
Jasmine	1	1	1	1	0

Exercise: Calculate TOT & Selection Bias

$$\begin{aligned}\text{TOT} &\equiv \mathbb{E}(Y_1 - Y_0 | D = 1) \\ &= \frac{1}{4}(1 + 1 + 0 + 0) = 0.5\end{aligned}$$

$$\begin{aligned}\text{SB} &\equiv \mathbb{E}(Y_0 | D = 1) - \mathbb{E}(Y_0 | D = 0) \\ &= \frac{1}{4}(0 + 0 + 1 + 1) \\ &\quad - \frac{1}{6}(0 + 1 + 1 + 1 + 0 + 1) \\ &= \frac{1}{2} - \frac{2}{3} = -\frac{1}{6} \approx -0.17\end{aligned}$$

Fundamental Decomposition in Disease Example

	D	Y	Y_0	Y_1	$(Y_1 - Y_0)$
Amir	1	1	0	1	1
Brooke	0	0	0	0	0
Carlos	0	1	1	0	-1
Derek	1	1	0	1	1
Elise	0	1	1	0	-1
Felix	1	1	1	1	0
Gabriela	0	1	1	0	-1
Hannah	0	0	0	0	0
Issac	0	1	1	0	-1
Jasmine	1	1	1	1	0

$$\underbrace{\mathbb{E}(Y|D=1) - \mathbb{E}(Y|D=0)}_{\text{Observed Difference of Means}=1/3} = \underbrace{\mathbb{E}(Y_1 - Y_0|D=1)}_{\text{TOT}=1/2} + \underbrace{[\mathbb{E}(Y_0|D=1) - \mathbb{E}(Y_0|D=0)]}_{\text{Selection Bias}=(-1/6)}$$

Conditional Expectation and Independence

$$\mathbb{E}(W|X = x) \equiv \sum_{\text{all } w} w \cdot \mathbb{P}(W = w|X = x)$$

Exercise: If W and X are independent what can we say about $\mathbb{E}(W|X = x)$?

Conditional Expectation and Independence

$$\mathbb{E}(W|X=x) \equiv \sum_{\text{all } w} w \cdot \mathbb{P}(W=w|X=x)$$

Exercise: If W and X are independent what can we say about $\mathbb{E}(W|X=x)$?

$$\mathbb{E}(W|X=x) \equiv \sum_{\text{all } w} w \cdot \mathbb{P}(W=w|X=x) = \sum_{\text{all } w} w \cdot \frac{\mathbb{P}(W=w, X=x)}{\mathbb{P}(X=x)}$$

$$= \sum_{\text{all } w} w \cdot \frac{\mathbb{P}(W=w)\mathbb{P}(X=x)}{\mathbb{P}(X=x)} = \sum_{\text{all } w} w \cdot \mathbb{P}(W=w)$$

$$\equiv \mathbb{E}(W)$$

Randomization eliminates selection bias.

Independence[3]

- $W \perp\!\!\!\perp X$ is shorthand for "W is **statistically independent** of X."
- $W \perp\!\!\!\perp X \iff \mathbb{P}(W = w, X = x) = \mathbb{P}(W = w)\mathbb{P}(X = x)$ for all w and x.
- Statistical independence implies **conditional mean independence**

$$W \perp\!\!\!\perp X \implies \mathbb{E}[W|X = x] = \mathbb{E}[W] \quad \text{and} \quad \mathbb{E}[X|W = w] = \mathbb{E}[X]$$

Random Assignment: $D \perp\!\!\!\perp (Y_0, Y_1)$

$$\text{TOT} = \mathbb{E}(Y_1|D = 1) - \mathbb{E}(Y_0|D = 1) = \mathbb{E}(Y_1) - \mathbb{E}(Y_0) \equiv \text{ATE}$$

$$\text{Selection Bias} \equiv \mathbb{E}(Y_0|D = 1) - \mathbb{E}(Y_0|D = 0) = \mathbb{E}(Y_0) - \mathbb{E}(Y_0) = 0$$

[3] See chapter 2 of the notes, https://expl.ai/LXPVDDN and my blog post for more on independence.

Let's try assigning the treatment at random!

```
library(tidyverse)

person <- c('Amir', 'Brooke', 'Carlos', 'Derek', 'Elise', 'Felix',
            'Gabriela', 'Hannah', 'Issac', 'Jasmine')
y0 <- c(0, 0, 1, 0, 1, 1, 1, 0, 1, 1)
y1 <- c(1, 0, 0, 1, 0, 1, 0, 0, 0, 1)

potential_outcomes <- tibble(person, y0, y1)
```

```r
potential_outcomes
```

```
## # A tibble: 10 x 3
##    person      y0    y1
##    <chr>    <dbl> <dbl>
##  1 Amir         0     1
##  2 Brooke       0     0
##  3 Carlos       1     0
##  4 Derek        0     1
##  5 Elise        1     0
##  6 Felix        1     1
##  7 Gabriela     1     0
##  8 Hannah       0     0
##  9 Issac        1     0
## 10 Jasmine      1     1
```

```r
potential_outcomes |>
  summarize(ATE = mean(y1 - y0))
```

```
## # A tibble: 1 x 1
##      ATE
##    <dbl>
## 1   -0.2
```

```r
set.seed(1983)

treatments <- c(0, 0, 0, 0, 0, 1, 1, 1, 1, 1)

treatments
```

```
##  [1] 0 0 0 0 0 1 1 1 1 1
```

```r
treatments <- sample(treatments)

treatments
```

```
##  [1] 0 0 0 1 1 1 0 1 1 0
```

```r
potential_outcomes |>
  bind_cols(d = treatments) |>
  mutate(y = (1 - d) * y0 + d * y1)
```

```
## # A tibble: 10 x 5
##    person      y0    y1     d     y
##    <chr>    <dbl> <dbl> <dbl> <dbl>
##  1 Amir         0     1     0     0
##  2 Brooke       0     0     0     0
##  3 Carlos       1     0     0     1
##  4 Derek        0     1     1     1
##  5 Elise        1     0     1     0
##  6 Felix        1     1     1     1
##  7 Gabriela     1     0     0     1
##  8 Hannah       0     0     1     0
##  9 Issac        1     0     1     0
## 10 Jasmine      1     1     0     1
```

```r
potential_outcomes |>
  bind_cols(d = treatments) |>
  mutate(y = (1 - d) * y0 + d * y1) |>
  group_by(d) |>
  summarize(ybar = mean(y))
```

```
## # A tibble: 2 x 2
##       d  ybar
##   <dbl> <dbl>
## 1     0   0.6
## 2     1   0.4
```

```r
potential_outcomes |>
  bind_cols(d = treatments) |>
  mutate(y = (1 - d) * y0 + d * y1) |>
  group_by(d) |>
  summarize(ybar = mean(y)) |>
  pull(ybar)
```

```
## [1] 0.6 0.4
```

```r
potential_outcomes |>
  bind_cols(d = treatments) |>
  mutate(y = (1 - d) * y0 + d * y1) |>
  group_by(d) |>
  summarize(ybar = mean(y)) |>
  pull(ybar) |>
  diff()
```

```
## [1] -0.2
```

```r
run_experiment <- function() {

  treatments <- sample(c(0, 0, 0, 0, 0, 1, 1, 1, 1, 1))

  potential_outcomes |>
    bind_cols(d = treatments) |>
    mutate(y = (1 - d) * y0 + d * y1) |>
    group_by(d) |>
    summarize(ybar = mean(y)) |>
    pull(ybar) |>
    diff()
}
```

```r
run_experiment()
```

```
## [1] 0.2
```

```r
run_experiment()
```

```
## [1] -0.6
```

```r
run_experiment()
```

```
## [1] 0.2
```

```r
simulations <- replicate(5000, run_experiment())

mean(simulations)
```

```
## [1] -0.19528
```